IJESRT

INTERNATIONAL JOURNAL OF ENGINEERING SCIENCES & RESEARCH TECHNOLOGY

Contra *v*-Open Mappings

S. Balasubramanian[*1], C. Sandhya[2] and P. Aruna Swathi Vyjayanthi[3]

[*1]Department of Mathematics, Government Arts College(A), Karur, Tamilnadu, India

[2]Department of Mathematics, C.S.R. Sarma College, Ongole, Andhrapradesh, India

[3]Research Scholar, Dravidian University, Kuppam, Andhrapradesh,India

mani55682@rediffmail.com

Abstract

The aim of this paper is to introduce and study the concept of contra-*v*-open mappings and the interrelationship between other Contra-open maps.

Keywords: *v*-open set, v-open map, *v*-closed map, Contra-open map, Contra-pre open map and Contra *v*-open map.

AMS Classification: 54C10, 54C08, 54C05.

Introduction

Mappings plays an important role in the study of modern mathematics, especially in Topology and Functional analysis. Open mappings are one such mappings which are studied for different types of open sets by various mathematicians for the past many years. N.Biswas, discussed about semiopen mappings in the year 1970, A.S.Mashhour, M.E.Abd El-Monsef and S.N.El-Deeb studied preopen mappings in the year 1982 and S.N.El-Deeb, and I.A.Hasanien defind and studied about preclosed mappings in the year 1983. Further Asit kumar sen and P. Bhattacharya discussed about pre-closed mappings in the year 1993. A.S.Mashhour, I.A.Hasanein and S.N.El-Deeb introduced α-open and α-closed mappings in the year in 1983, F.Cammaroto and T.Noiri discussed about semipre-open and semipre-clsoed mappings in the year 1989 and G.B.Navalagi further verified few results about semipreclosed mappings. M.E.Abd El-Monsef, S.N.El-Deeb and R.A.Mahmoud introduced β-open mappings in the year 1983 and Saeid Jafari and T.Noiri, studied about β-closed mappings in the year 2000. C. W. Baker, introduced Contra-open functions and contra-closed functions in the year 1997. M.Caldas and C.W.Baker introduced contra pre-semiopen Maps in the year 2000. In the year 2010, S. Balasubramanian and P.A.S.Vyjayanthi introduced *v*-open mappings and in the year 2011 they further defined almost *v*-open mappings. In the last year S. Balasubramanian and P.A.S.Vyjayanthi introduced *v*-closed and Almost *v*-closed mappings. Inspired with these concepts and its interesting properties we in this paper tried to study a new variety of open maps called contra *v*-open maps. Throughout the paper *X*, *Y* means topological spaces (X, τ) and (Y,σ) on which no separation axioms are assured.

Preliminaries

Definition 1: $A\subseteq X$ is said to be

a) regular open[pre-open; semi-open; α-open; β-open] if $A = int(cl(A))$ [$A \subseteq int(cl(A))$; $A\subseteq cl(int(A))$; $A\subseteq int(cl(int(A)))$; $A\subseteq cl(int(cl(A)))$] and regular closed[pre-closed; semi-closed; α-closed; β-closed] if $A = cl(int(A))$[$cl(int(A)) \subseteq A$; $int(cl(A))\subseteq A$; $cl(int(cl(A)))\subseteq A$; $int(cl(int(A)))\subseteq A$]

b) *v*-open if there exists regular-open set U such that $U\subseteq A\subseteq cl(U)$.

c) g-closed[rg-closed] if $cl(A)\subset U$[$rcl(A)\subset U$] whenever $A\subset U$ and U is open[r-open] in X and g-open[rg-open] if its complement X - A is g-closed[rg-closed].

Remark 1: We have the following implication diagrams for open sets.

r-open set → rα-open set → *v*-open set
↓ ↓
pre-open set ← open set → α-open set → semi-open set → β-open set.

Definition 2: A function $f:X\rightarrow Y$ is said to be

a) continuous [resp: semi-continuous, r-continuous, v-continuous] if the inverse image of every open set is open. [resp: semi open, regular open, v--open].
b) irresolute [resp: r-irresolute, v-irresolute] if the inverse image of every semi open [resp: regular open, v-open] set is semi open. [resp: regular open, v-open].
c) closed [resp: semi-closed, r-closed] if the image of every closed set is closed [resp: semi closed, regular closed].
d) g-continuous [resp: rg-continuous] if the inverse image of every closed set is g-closed. [resp: rg-closed].
e) contra open[resp: contra semi-open; contra pre-open; contra rα-open] if the image of every open set in X is closed[resp: semi-closed; pre-closed; rα-closed] in Y.

Definition 3: X is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

Contra *v*-Open Mappings

Definition 1: A function $f: X\rightarrow Y$ is said to be Contra v-open if the image of every open set in X is v-closed in Y.

Theorem 1: Every Contra $r\alpha$-open map is Contra v-open but not conversely.

Proof: Let A$\subseteq X$ be open $\Rightarrow f$(A) is $r\alpha$-closed in Y since $f:X\rightarrow Y$ is Contra $r\alpha$-open $\Rightarrow f$(A) is v-closed in Y since every rα-closed set is v-closed. Hence f is Contra v-open.

Example 1: Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f:X\rightarrow Y$ be defined f(a) = b, f(b) = c and f(c) = a. Then f is Contra v-open, Contra semi-open, Contra rα-open and Contra β-open but not Contra open, Contra pre-open, Contra r-open, Contra α-open and Contra rp-open.

Example 2: Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\phi, \{a, c\}, Y\}$. Let $f:X\rightarrow Y$ be defined f(a) = c, f(b) = b and f(c) = a. Then f is Contra pre-open, Contra rp-open and Contra β-open but not Contra open, Contra semi-open, Contra r-open, Contra v-open, Contra α-open and Contra rα-open.

Theorem 2: Every Contra r-open map is Contra v-open but not conversely.

Proof: Let A$\subseteq X$ be open $\Rightarrow f$(A) is r-closed in Y since $f:X\rightarrow Y$ is Contra r-open $\Rightarrow f$(A) is v-closed in Y since every rα-closed set is v-closed. Hence f is Contra v-open.

Example 3: Let $X = Y = \{a, b, c\}$; $\tau = \{\phi, \{a\}, X\}$; $\sigma = \{\phi, \{a\}, \{a, b\}, Y\}$. Let $f:X\rightarrow Y$ be defined f(a) = c, f(b) = b and f(c) = a. Then f is Contra open, Contra semi-open, Contra pre-open, Contra β-open, Contra α-open and Contra rα-open but not Contra r-open, Contra v-open and Contra rp-open.

Theorem 3: Every Contra v-open map is Contra semi-open but not conversely.

Proof: Let A$\subseteq X$ be open $\Rightarrow f$(A) is v-closed in Y since $f:X\rightarrow Y$ is Contra v-open $\Rightarrow f$(A) is semi-closed in Y since every v-closed set is semi-closed. Hence f is Contra semi-open.

Theorem 4: Every Contra v-open map is Contra β-open but not conversely.

Proof: Let A$\subseteq X$ be open $\Rightarrow f$(A) is v-closed in Y since $f:X\rightarrow Y$ is Contra v-open $\Rightarrow f$(A) is β-closed in Y since every v-closed set is β-closed. Hence f is Contra β-open.

Note 1:

a) Contra open maps and Contra v-open maps are independent of each other.
b) Contra α-open map and Contra v-open map are independent of each other.
c) Contra pre open map and Contra v-open map are independent of each other.

Note 2: We have the following implication diagram among the open maps.

Contra r-open→ Contra rα-open → Contra v-open
↓ ↓
Contra pre-open ← Contra open → Contra α-open → Contra-semi-open → Contra β-open.

None is reversible.

Theorem 5: If RαC(Y) = vC(Y) then f is Contra rα-open iff f is Contra v-open.

Proof: Follows from theorem 3.1

Conversely Let A$\subseteq X$ be open $\Rightarrow f$(A) is v-closed in Y since $f:X\rightarrow Y$ is Contra v-open $\Rightarrow f$(A) is $r\alpha$-closed in Y since every v-closed set is $r\alpha$-closed. Hence f is Contra $r\alpha$-open.

Theorem 6: If vC(Y) = RC(Y) then f is Contra r-open iff f is Contra v-open.

Proof: Follows from theorem 3.2

Conversely Let A$\subseteq X$ be open $\Rightarrow f$(A) is v-closed in Y since $f:X\rightarrow Y$ is Contra v-open $\Rightarrow f$(A) is r-closed in Y since every v-closed set is r-closed. Hence f is Contra r-open.

Theorem 7: If vC(Y) = αC(Y) then f is Contra α-open iff f is Contra v-open.

Proof: Let A$\subseteq X$ be open $\Rightarrow f$(A) is α-closed in Y since $f:X\rightarrow Y$ is Contra α-open $\Rightarrow f$(A) is v-closed in Y since every α-closed set is v-closed. Hence f is Contra v-open.

Conversely Let $A \subseteq X$ be open $\Rightarrow f(A)$ is v-closed in Y since $f:X \to Y$ is Contra v-open $\Rightarrow f(A)$ is α-closed in Y since every v-closed set is α-closed. Hence f is Contra α-open.

Theorem 8: If f is open[r-open] and g is Contra v-open then $g \ o \ f$ is Contra v-open.

Proof: Let $A \subseteq X$ be open $\Rightarrow f(A)$ is open in $Y \Rightarrow g(f(A))$ is v-closed in $Z \Rightarrow g \ o \ f(A)$ is v-closed in Z. Hence $g \ o \ f$ is Contra v-open.

Theorem 9: If f is open[r-open] and g is Contra r-open then $g \ o \ f$ is Contra v-open.

Proof: Let $A \subseteq X$ be open $\Rightarrow f(A)$ is open in $Y \Rightarrow g(f(A))$ is r-closed in $Z \Rightarrow g \ o \ f(A)$ is v-closed in Z. Hence $g \ o \ f$ is Contra v-open.

Theorem 10: If f is open[r-open] and g is Contra $r\alpha$-open then $g \ o \ f$ is Contra v-open.

Proof: Let $A \subseteq X$ be open in X $\Rightarrow f(A)$ is open in Y $\Rightarrow g(f(A))$ is $r\alpha$-closed in Z $\Rightarrow g(f(A))$ is v-closed in Z $\Rightarrow g \bullet f(A)$ is v-closed in Z. Hence $g \bullet f$ is almost Contra v-open.

Corollary 1.1:

a) If f is open[r-open] and g is Contra v-open then $g \ o \ f$ is Contra semi-open and hence Contra β-open.

b) If f is open[r-open] and g is Contra r-open then $g \ o \ f$ is Contra semi-open and hence Contra β-open.

c) If f is open[r-open] and g is Contra $r\alpha$-open then $g \ o \ f$ is Contra semi-open and hence Contra β-open.

Theorem 14: If f is Contra open[Contra-r-open] and g is v-closed then $g \ o \ f$ is Contra-v-open.

Proof: Let $A \subseteq X$ be open in X $\Rightarrow f(A)$ is closed in Y $\Rightarrow g(f(A))$ is v-closed in Z $\Rightarrow g \bullet f(A)$ is v-closed in Z. Hence $g \bullet f$ is Contra v-open.

Theorem 15: If f is Contra open[Contra-r-open] and g is r-closed then $g \ o \ f$ is Contra-v-open.

Proof: Let $A \subseteq X$ be open in X $\Rightarrow f(A)$ is closed in Y $\Rightarrow g(f(A))$ is r-closed in Z $\Rightarrow g(f(A))$ is v-closed in Z $\Rightarrow g \bullet f(A)$ is v-closed in Z. Hence $g \bullet f$ is Contra v-open.

Theorem 16: If f is Contra open[Contra-r-open] and g is $r\alpha$-closed then $g \ o \ f$ is Contra-v-open.

Proof: Let $A \subseteq X$ be open in X $\Rightarrow f(A)$ is closed in Y $\Rightarrow g(f(A))$ is $r\alpha$-closed in Z $\Rightarrow g(f(A))$ is v-closed in Z $\Rightarrow g \bullet f(A)$ is v-closed in Z. Hence $g \bullet f$ is Contra v-open.

Corollary 1.2:

a) If f is Contra open[Contra-r-open] and g is v-closed then $g \ o \ f$ is Contra-semi-open and hence Contra β-open.

b) If f is Contra open[Contra-r-open] and g is r-closed then $g \ o \ f$ is Contra-semi-open and hence Contra β-open.

c) If f is Contra open[Contra-r-open] and g is $r\alpha$-clsoed then $g \ o \ f$ is Contra-semi-open and hence Contra β-open.

Theorem 20: If $f: X \to Y$ is Contra v-open then $v(\overline{f(\bar{A})}) \subseteq f(\bar{A})$

Proof: Let $A \subseteq X$ and $f: X \to Y$ be Contra v-open. Then $f(\bar{A})$ is v-closed in Y and $f(A) \subseteq f(\bar{A})$. This implies $v\overline{(f(A))} \subseteq v(\overline{f(\bar{A})})$ $\rightarrow$ (1)

Since $f(\bar{A})$ is v-open in Y, $v(\overline{f(\bar{A})}) = f(\bar{A})$ $\rightarrow$ (2)

Using (1) & (2) we have $v(\overline{f(\bar{A})}) = f(\bar{A})$ for every subset A of X.

Remark 2: Converse is not true in general as shown by the following example.

Example 4: Let $X = Y = \{a, b, c\}$ $\tau = \{\phi, \{a\}, \{a, b\}, X\}$ $\sigma = \{\phi, \{a\}, \{b\}, \{a, b\}, Y\}$. Let $f: X \to Y$ be the identity map. Then $v\overline{(f(A))} \subseteq f(\bar{A})$ for every subset A of X. But f is not Contra v-open since $f(\{a, b\}) = \{a, b\}$ is not v-closed.

Corollary 1.3: If $f:X \to Y$ is Contra r-open then $v(\overline{f(\bar{A})}) \subseteq f(\bar{A})$.

Theorem 21: If $f:X \to Y$ is Contra v-open and $A \subseteq X$ is open, $f(A)$ is τ_v-closed in Y.

Proof: Let $A \subseteq X$ and $f:X \to Y$ be Contra v-open $\Rightarrow v(\overline{f(\bar{A})}) \subseteq f(\bar{A})$ (by theorem 3.20.) $\Rightarrow v(\overline{f(\bar{A})}) \subseteq f(A)$ since $f(A) = f(\bar{A})$ as A is open. But $f(A) \subseteq v(\overline{f(\bar{A})})$. Therefore we have $f(A) = v(\overline{f(\bar{A})})$. Hence $f(A)$ is τ_v-closed in Y.

Corollary 1.4: If $f:X \to Y$ is Contra r-open, then $f(A)$ is τ_v-closed in Y if A is r-open set in X.

Theorem 22: If $v((\bar{A})) = r((\bar{A}))$ for every $A \subseteq Y$ then the following are equivalent.

a) $f:X \to Y$ is Contra v-open map.

b) $v(\overline{f(\bar{A})}) \subseteq f(\bar{A})$.

Proof: (a) $\Rightarrow$(b) follows from theorem 3.20.

(b) $\Rightarrow$(a) Let A by any open set in X.

Then $f(A) = f(\bar{A}) \supset v\overline{(f(A))}$ [by hypothesis] We have $f(A) \subseteq v(\overline{f(\bar{A})})$. Combining these two we have, $f(A) = v(\overline{f(\bar{A})}) = r(\overline{f(A)})$ (by given condition) which implies $f(A)$ is r-open and hence $f(A)$ is v-closed. Thus for every open set A in X, we have $f(A)$ is v-closed in Y. Therefore f is Contra v-open.

Theorem 23: $f:X \to Y$ is Contra v-open iff for each subset S of Y and each closed set U containing $f^{-1}(S)$, there is an v-open set V of Y such that $S \subseteq V$ and $f^{-1}(V) \subseteq U$.

Proof: Assume $f{:}X\rightarrow Y$ is contra v-open. Let S $\subseteq Y$ and U be a closed set of X containing f^{-1}(S). Then X-U is open in X and $f(X$-U) is v-closed in Y as f is contra v-open and V=Y- $f(X$-U) is v-open in Y. f^{-1}(S) $\subseteq$ U $\Rightarrow$ S$\subseteq f$(U) $\Rightarrow$ S $\subseteq$ V and f^{-1}(V) = f^{-1}(Y- $f(X$-U)) = $f^{-1}(Y)$- f^{-1}(f (X-U)) = $f^{-1}(Y)$-(X-U) = X-(X-U) = U

Conversely Let F be open in X $\Rightarrow$ F^c is closed. Then $f^{-1}(f(F^c)) \subseteq F^c$.
By hypothesis there exists an v-open set V of Y, such that $f(F^c) \subseteq$ V and $f^{-1}(V) \supset F^c$ and so F $\subseteq [f^{-1}(V)]^c$. Hence $V^c \subseteq f(F) \subseteq f[f^{-1}(V)^c] \subseteq V^c \Rightarrow f(F) \subseteq V^c \Rightarrow f(F)=V^c$. Thus f (F) is v-closed in Y. Therefore f is Contra v-open.
Remark 3: Composition of two Contra v-open maps is not Contra v-open in general.
Theorem 24: Let X, Y, Z be topological spaces and every v-closed set is open[r-open] in Y. Then the composition of two Contra v-open[Contra r-open] maps is Contra v-open.
Proof: (a) Let $f{:}X\rightarrow Y$ and g:$Y\rightarrow Z$ be Contra v-open maps. Let A be any open set in X
$\Rightarrow f$ (A) is v-closed in $Y\Rightarrow f$ (A) is open in Y (by assumption) $\Rightarrow g(f$ (A)) is v-closed in $Z \Rightarrow g$ *o* f(A) is v-closed in Z. Therefore g *o* f is Contra v-open.
(b) Let $f{:}X\rightarrow Y$ and g:$Y\rightarrow Z$ be Contra v-open maps. Let A be any open set in X
$\Rightarrow f$ (A) is r-closed in $Y \Rightarrow f$ (A) is v-closed in $Y\Rightarrow f$ (A) is r-open in Y (by assumption) $\Rightarrow f$ (A) is open in Y (by assumption) $\Rightarrow g(f$ (A)) is r-closed in Z $\Rightarrow g$ *o* f(A) is v-closed in Z. Therefore gof is Contra v-open.
Theorem 25: Let X, Y, Z be topological spaces and Y is discrete topological space in Y. Then the composition of two Contra v-open[Contra r-open] maps is Contra v-open.
Theorem 26: If $f{:}X\rightarrow Y$ is g-open, g:$Y\rightarrow Z$ is Contra v-open [Contra r-open] and Y is $T_{1/2}$ [r-$T_{1/2}$] then g *o* f is Contra v-open.
Proof: (a) Let A be an open set in X. Then f(A) is g-open set in $Y \Rightarrow f$ (A) is open in Y as Y is $T_{1/2} \Rightarrow g(f$ (A)) is v-closed in Z since g is Contra v-open $\Rightarrow g$ *o* f (A) is v-closed in Z. Hence g *o* f is Contra v-open.
(b) Let A be an open set in X. Then f(A) is g-open set in $Y \Rightarrow f$ (A) is open in Y as Y is $T_{1/2} \Rightarrow g(f$ (A)) is r-closed in Z since g is Contra r-open $\Rightarrow g$ *o* f (A) is v-closed in Z. Hence gof is Contra v-open.
Corollary 1.5: If $f{:}X\rightarrow Y$ is g-closed, $g{:}Y\rightarrow Z$ is Contra v-open [Contra r-open] and Y is $T_{1/2}$ [r-$T_{1/2}$] then gof is Contra p-open and hence Contra β-open.
Theorem 27: If $f{:}X\rightarrow Y$ is rg-closed, $g{:}Y\rightarrow Z$ is Contra v-open [Contra r-open] and Y is r-$T_{1/2}$, then g *o* f is Contra v-open.
Proof: Let A be an open set in X. Then f(A) is rg-open in Y $\Rightarrow f$(A) is r-open in Y since Y is r-$T_{1/2}$ $\Rightarrow f$(A) is open in Y since every r-open set is open $\Rightarrow$ g(f(A)) is v-closed in $Z \Rightarrow$ g o f(A) is v-closed in Z. Hence gof is Contra v-open.
Corollary 1.6: If $f{:}X\rightarrow Y$ is rg-closed, g:$Y\rightarrow Z$ is Contra v-open [Contra r-open] and Y is r-$T_{1/2}$, then g *o* f is Contra semi-open and hence Contra β-open.
Theorem 28: If $f{:}X\rightarrow Y$, $g{:}Y\rightarrow Z$ be two mappings such that gof is Contra v-open [Contra r-open] then the following statements are true.
a) If f is continuous [r-continuous] and surjective then g is Contra v-open.
b) If f is g-continuous, surjective and X is $T_{1/2}$ then g is Contra v-open.
c) If f is rg-continuous, surjective and X is r-$T_{1/2}$ then g is Contra v-open.
Proof: (a) Let A be an open set in $Y \Rightarrow f^{-1}$(A) is open in $X \Rightarrow (g\ o\ f)(f^{-1}$(A)) is v-closed in $Z \Rightarrow g$(A) is v-closed in Z. Hence g is Contra v-open.
(b) Let A be an open set in $Y \Rightarrow f^{-1}$(A) is g-open in $X \Rightarrow f^{-1}$(A) is open in X[since X is $T_{1/2}$] $\Rightarrow (g\ o\ f)(f^{-1}$(A)) is v-closed in $Z \Rightarrow g$(A) is v-closed in Z. Hence g is Contra v-open.
(c) Let A be an open set in Y. $\Rightarrow f^{-1}$(A) is g-open in $X \Rightarrow f^{-1}$(A) is open in X[since X is r-$T_{1/2}$] $\Rightarrow (g\ o\ f)(f^{-1}$(A)) is v-closed in $Z \Rightarrow g$(A) is v-closed in Z. Hence g is Contra v-open.
Corollary 1.7: If $f{:}X\rightarrow Y$, g:$Y\rightarrow Z$ be two mappings such that gof is Contra v-open [Contra r-open] then the following statements are true.
a) If f is continuous [r-continuous] and surjective then g is Contra semi-open and hence Contra β-open.
b) If f is g continuous, surjective and X is $T_{1/2}$ then g is Contra semi-open and hence Contra β-open.
c) If f is rg-continuous, surjective and X is r-$T_{1/2}$ then g is Contra semi-open and hence Contra β-open.
Theorem 29: If $f{:}X\rightarrow Y$ is Contra v-open and A is an open set of X then $f_A{:}(X, \tau(A)) \rightarrow (Y, \sigma)$ is Contra v-open.
Proof: (a) Let F be an open set in A. Then F = A$\cap$E for some open set E of X and so F is open in $X \Rightarrow f$(A) is v-closed in Y. But f(F) = f_A(F). Therefore f_A is Contra v-open.

Theorem 30: If $f:X\rightarrow Y$ is Contra r-open and A is an open set of X then $f_A:(X, \tau(A)) \rightarrow (Y, \sigma)$ is Contra v-open.
Proof: Let F be an open set in A. Then F = A$\cap$E for some open set E of X and so F is open in $X \Rightarrow f(A)$ is r-closed in $Y \Rightarrow f(A)$ is v-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is Contra v-open.
Corollary 1.8: If $f:X\rightarrow Y$ is Contra v-open [Contra r-open] and A is an open set of X then $f_A:(X, \tau(A)) \rightarrow (Y, \sigma)$ is Contra semi-open and hence Contra β-open.
Theorem 31: If $f:X\rightarrow Y$ is Contra v-open, X is $T_{1/2}$ and A is g-open set of X then $f_A:(X, \tau(A)) \rightarrow (Y, \sigma)$ is Contra v-open.
Proof: Let F be an open set in A. Then F = A$\cap$E for some open set E of X and so F is open in $X \Rightarrow f(A)$ is v-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is Contra v-open.
Theorem 32: If $f:X\rightarrow Y$ is Contra-r-open, X is $T_{1/2}$ and A is g-open set of X then $f_A:(X, \tau(A)) \rightarrow (Y, \sigma)$ is Contra v-open.
Proof: Let F be an open set in A. Then F = A$\cap$E for some open set E of X and so F is open in $X \Rightarrow f(A)$ is r-closed in Y. $\Rightarrow f(A)$ is v-closed in Y. But $f(F) = f_A(F)$. Therefore f_A is Contra v-open.
Corollary 1.9: If $f:X\rightarrow Y$ is Contra v-open [Contra r-open], X is $T_{1/2}$, A is g-open set of X then $f_A:(X, \tau(A)) \rightarrow (Y, \sigma)$ is Contra semi-open and hence Contra β-open.
Theorem 33: If f_i: $X_i\rightarrow Y_i$ be Contra v-open [Contra r-open] for i =1, 2. Let f: $X_1xX_2\rightarrow Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1),f_2(x_2))$. Then $f:X_1xX_2\rightarrow Y_1xY_2$ is Contra v-open.
Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is open in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is v-closed set in Y_1xY_2. Then $f(U_1xU_2)$ is v-closed set in Y_1xY_2. Hence f is Contra v-open.
Corollary 1.10: If f_i: $X_i\rightarrow Y_i$ be Contra v-open [Contra r-open] for i =1,2. Let $f:X_1xX_2\rightarrow Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1),f_2(x_2))$, then $f:X_1xX_2\rightarrow Y_1xY_2$ is Contra semi-open and hence Contra β-open.
Theorem 34: Let $h:X\rightarrow X_1xX_2$ be Contra v-open. Let $f_i:X\rightarrow X_i$ be defined as $h(x)=(x_1,x_2)$ and $f_i(x) = x_i$. Then f_i: $X\rightarrow X_i$ is Contra v-open for i =1, 2.
Proof: Let U_1 be open in X_1, then U_1x X_2 is open in X_1x X_2, and $h(U_1x$ $X_2)$ is v-closed in X. But $f_1(U_1) = h(U_1x$ $X_2)$, therefore f_1 is Contra v-open. Similarly we can show that f_2 is also Contra v-open and thus f_i: X $\rightarrow X_i$ is Contra v-open for i = 1, 2.
Corollary 1.11: Let h: $X\rightarrow X_1xX_2$ be Contra v-open. Let f_i: $X\rightarrow X_i$ be defined as $h(x) = (x_1,x_2)$ and $f_i(x) = x_i$. Then f_i: $X\rightarrow X_i$ is Contra semi-open and hence Contra β-open for i =1,2.

Conclusion

In this paper we introduced the concept of contra v-open mappings, studied their basic properties and the interrelationship between other open maps.

References

[1] Asit Kumar Sen and Bhattcharya.P., On preclosed mappings, Bull.Cal.Math.Soc.,85, 409-412(1993).
[2] Baker.C.W., Contra-open functions and contra-closed functions, Math. Today (Ahmedabad) 15 (1997), 19–24.
[3] Balasubramanian.S., and Vyjayanthi.P.A.S., v-open Mappings – Scientia Magna Vol 6. No. 4(2010) 118 – 124.
[4] Balasubramanian.S., and Vyjayanthi.P.A.S., v-closed Mappings – Jr. Advanced Research in Pure Mathematics, Vol 3. No. 1(2011)135 – 143
[5] Balasubramanian.S., vg-open mappings – Inter. J. Comp. Math. Sci. and Application Vol.5. No.2(2011)7-14.
[6] Balasubramanian.S., and Krishnamurthy.T.K., Regular pre-Closed mappings – Inter. J. Math. Archive, Vol 2, No. 8(2011) 1411 – 1415.
[7] Balasubramanian.S., Vyjayanthi.P.A.S., and Sandhya.C., Almost v-closed Mappings – Inter. J. Math. Archive,Vol 2, No. 10 (2011) 1920 – 1925.
[8] Balasubramanian.S., Sandhya.C., and Vyjayanthi.P.A.S., Almost v-open Mappings – Inter. J. Math. Archive,Vol 2, No.10 (2011) 1943 – 1948.
[9] Balasubramanian.S., and Lakshmi Sarada.M., *gpr*-closed and *gpr*-open functions- International Journal of Mathematical Engineering and Science, Vol.1,No.6(2012)09 – 16.
[10] Balasubramanian.S., Somewhat M-*vg*-open map, Aryabhatta J. Math and Informatics, Vol.4,No.2(2012)315 – 320
[11] Balasubramanian.S., and Lakshmi Sarada.M., Almost *gpr*-closed and Almost *gpr*-open functions- Aryabhatta Journal of Mathematics and Informatics (In press)
[12] Caldas,M., and Baker,C.W., Contra Pre-semiopen Maps, Kyungpook Math.Journal, 40 (2000), 379 – 389.

[13] Di Maio. G., and Noiri.T, I. J. P. A. M., 18(3) (1987) 226-233.
[14] Dontchev.J., Mem.Fac.Sci.Kochi Univ.ser.A., Math., 16(1995), 35-48.
[15] Dunham.W., $T_{1/2}$ Spaces, Kyungpook Math. J.17(1977), 161-169.
[16] Long.P.E., and Herington.L.L., Basic Properties of Regular Closed Functions, Rend. Cir. Mat. Palermo, 27(1978), 20-28.
[17] Malghan.S.R., Generalized closed maps, J. Karnatak Univ. Sci., 27(1982), 82 -88.
[18] Mashour.A.S., Hasanein.I.A., and El.Deep.S.N., α-continuous and α-open mappings, Acta Math. Hungar.,41(1983), 213-218.
[19] Noiri.T., A generalization of closed mappings, Atti. Accad. Naz. Lincei Rend. Cl. Sci. Fis. Mat. Natur., 54 (1973),412-415.
[20] Noiri.T., Almost αg-closed functions and separation axioms, Acta Math. Hungar. 82(3) (1999),193-205.
[21] Palaniappan.N., Studies on Regular-Generalized Closed Sets and Maps in Topological Spaces, Ph. D Thesis, Alagappa University, Karikudi, (1995).
[22] Vadivel.A., and Vairamanickam.K., $rg\alpha$-Closed and $rg\alpha$-open maps in topological spaces. Int.Journal of Math. Analysis, 2010, 4(10): 453 - 468.